6-2 Day 2-Integration by U-Substitution

Learning Objectives:

I can evaluate an integral using u-substitution.

Ex1. Evaluate

1.) $\int (2x+1)^5\,dx$

$(2x+1)^6$

$6\square \cdot 2 = 1$

$\frac{1}{12} = \square$

$u = 2x + 1$

$\frac{du}{dx} = \frac{2}{1}$

$2dx = du$

$dx = \frac{du}{2}$

$= \int u^5 \cdot \frac{du}{2}$

$= \int \frac{1}{2} u^5 du$

$\frac{1}{2} \cdot \frac{1}{6} u^6 + C$

$= \frac{1}{12} u^6 + C$

$= \frac{1}{12} (2x+1)^6 + C$

2.) $\int \sqrt{3x+8}\,dx$

$u = 3x+8$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$\int u^{1/2} \cdot \frac{du}{3}$

$= \int \frac{1}{3} u^{1/2}\,du$

$= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C$

$= \frac{2}{9}(3x+8)^{3/2} + C$

3.) $\int \sin\left(3x+\frac{\pi}{2}\right)dx$

$u = 3x + \frac{\pi}{2}$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$= \int \sin(u) \cdot \frac{du}{3}$

$= -\frac{1}{3}\cos(u) + c$

$= -\frac{1}{3}\cos\left(3x+\frac{\pi}{2}\right) + c$

Dec 11-12:35 PM

4.) $\int x\left(\sqrt[3]{2x^2+5}\right)dx$ $= \int x\sqrt[3]{u}\cdot\frac{du}{4x}$

$u = 2x^2+5$

$\frac{du}{dx} = 4x$

$dx = \frac{du}{4x}$

$= \int \frac{1}{4}u^{1/3}\,du$

$= \frac{1}{4}\cdot\frac{3}{4}u^{4/3}+C$

$= \frac{3}{16}(2x^2+5)^{4/3}+C$

Dec 11-12:35 PM

5.) $\int (3x^2+6x+2)^2(x+1)dx$

$u = 3x^2+6x+2$

$\frac{du}{dx} = 6x+6$

$dx = \frac{du}{6x+6}$

$dx = \frac{du}{6(x+1)}$

$\int \cancel{(x+1)} \cdot u^2 \cdot \frac{du}{6\cancel{(x+1)}}$

$\int \frac{1}{6} \cdot u^2 \cdot du$

$\frac{1}{3} \cdot \frac{1}{6} \cdot u^3 + C$

$\frac{1}{18}(3x^2+6x+2)^3 + C$

Dec 11-12:35 PM

6.) $\int \frac{3x-4}{3x^2-8x+3}\,dx$

$u = 3x^2 - 8x + 3$

$\frac{du}{dx} = 6x - 8$

$dx = \frac{du}{2(3x-4)}$

$\int \frac{\cancel{3x-4}}{2\cancel{(3x-4)}} \cdot u^{-1}\,du$

$\frac{1}{2}\ln u + C$

$\frac{1}{2}\ln(3x^2-8x+3) + C$

Dec 11-12:35 PM

7.) $\int \tan x dx$

$= \int \frac{\sin x}{\cos x} dx = \int \frac{\cancel{\sin x}}{u} \cdot \frac{du}{-\cancel{\sin x}}$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$= \int -\frac{1}{u} du$

$= -\ln u + C$

$= -\ln(\cos x) + C$

8.) $\int \cos^2(2x)\sin(2x)dx$

$= \int (\cos(2x))^2 \sin(2x)dx$

$u = \cos(2x)$

$\frac{du}{dx} = -2\sin 2x$

$dx = \frac{du}{-2\sin 2x}$

$\int u^2 \sin 2x \cdot \frac{du}{-2\sin 2x}$

$\int u^2 \cdot -\frac{du}{2}$

$+\frac{1}{3}u^3 \cdot -\frac{1}{2} + C = -\frac{1}{6}u^3 + C$

$= -\frac{1}{6}\cos^3(2x) + C$

9.) $\int_{e}^{e^3} \frac{\ln x}{x} dx$

$u = \ln x$ $\quad \ln e^3 = 3$ $\quad \ln e = 1$

$\frac{du}{dx} = 1/x = \quad dx = du\,x$

$\int \frac{u\, du\, \cancel{x}}{\cancel{x}} = \int u\, du$

$1/2\, u^2 = 1/2 \ln^2 x \Big|_{e}^{e^3}$

$\frac{1}{2}(\ln e^3)^2 - \frac{1}{2}(\ln e)^2$

$\frac{1}{2} \cdot 9 - \frac{1}{2} \cdot 1$

4

$\frac{1}{2} u^2 \Big|_1^3 = \frac{9}{2} - \frac{1}{2}$

$= 4$

Ex2. If you use the u-substitution u=lnx, which of the following integrals is

equivalent to $\int_e^{e^3} \frac{\ln x}{x} dx$

a.) $\int_e^{e^3} u\,du$

b.) $\int_1^{3} u\,du$

c.) $\int_e^{e^3} \frac{1}{u} du$

d.) $\int_1^{3} \frac{1}{u} du$

e.) $\int_e^{e^3} \ln u\,du$

Dec 11-12:35 PM

Homework

pg 337 # 18, 20, 24, 25, 27, 28, 33, 35, 37, 38, 40, 41, 44, 47, 49, 53, 58, 65, 66, 71, 73, 74, 76